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THE EFFECT OF BROKEN STRINGERS ON THE STRESS INTENSITY FACTOR FOR A UNIFORMLY STIFFENED SHEET CONTAINING A CRACK

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16. Abstract

A linear elastic stress analysis was made of a centrally cracked sheet stiffened by riveted, uniformly spaced and sized stringers. The stress intensity factor for the sheet and the load concentration factor for the most highly loaded stringer were determined for various numbers of broken stringers. A broken stringer causes the stress intensity factor to be very high when the crack tip is near the broken stringer but causes little effect when the crack tip extends beyond several intact stringers. A broken stringer also causes an increase in the load concentration factor of the adjacent stringers.

The calculated residual strengths and fatigue-crack-growth lives of a stiffened aluminum sheet with a broken stringer were only slightly less than a sheet with all intact stringers and were still much higher than those of an unstiffened sheet. The strengths and lives with a broken stringer were also higher for stiffer and more closely spaced stringers and rivets — much as when all stringers are intact.

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ABSTRACT

The stress intensity factor for the sheet and the force in the most highly loaded stringer were determined for a centrally cracked sheet stiffened by riveted, uniformly spaced and sized stringers of which various numbers were assumed to be broken. Also determined were the effects of stringer stiffness, stringer spacing, and rivet spacing.

The principal results of the analysis are:

1. Broken stringers cause higher stress intensity factors, especially when the crack tip is near a broken stringer.
2. Once the crack tip extends beyond several intact stringers, the stress intensity factor is no longer strongly affected by broken stringers, and it is reduced by the use of stiffer or more closely spaced stringers or smaller rivet spacing.
3. Broken stringers cause a higher load concentration in the adjacent stringers. The load concentration is lower for stiffer stringers.

Residual strengths and fatigue-crack-growth lives were calculated for a stiffened 2024-T3 sheet, with and without a broken stringer, and an unstiffened sheet. The sheet was assumed to fail before the remaining stringers. The strengths and lives for the stiffened sheet with a broken stringer were only slightly lower than those without a broken stringer, and were much higher than those for an unstiffened sheet. They were also higher for stiffer or more closely spaced stringers or more closely spaced rivets — much as when all stringers are intact.

INTRODUCTION

To comply with fail-safe and damage tolerance design criteria for aircraft structure, design methods are required to predict fatigue-crack-growth life and residual strength of cracked and damaged structure. Sheet-stringer type construction is widely used in aircraft and is generally regarded as a redundant type of construction that has inherent fail-safe and damage tolerant capabilities. In a previous investigation (ref. 1), the stress intensity factor for the sheet and the forces in the most highly loaded rivet and stringer were determined for a centrally cracked sheet with riveted and uniformly spaced stringers.

Because both sheet and stringer can develop cracks at a fastener hole or be damaged accidentally, multiple cracks or failures must be taken into account. Consequently, in the present investigation, the stress intensity factor for the sheet and the force in the most highly loaded stringer were calculated for a stiffened sheet with various numbers of broken stringers. The effects of systematic variations in stringer stiffness, stringer spacing, and rivet spacing were also determined. The present analysis method was that used in reference 1, but modified to account for the broken stringers.

To interpret the results, the stress intensity factor was used to calculate residual strengths and fatigue-crack-growth lives for a stiffened panel with and without broken stringers.

LIST OF SYMBOLS

A_{ij}^{s}	influence coefficient
$\bar{A}_{ij}^{s}$	influence coefficient
a	half-length of crack
B_{i}^{s}	influence coefficient
$\bar{B}_{i}^{s}$	influence coefficient
b	stringer spacing
b_o	specific value of stringer spacing
d	rivet diameter
E	Young's modulus of elasticity
F_{tu}	ultimate tensile strength
K	stress intensity factor
K_c	critical stress intensity factor
L	stringer load concentration factor
M_b	number of broken stringers
N	number of rivets
p	rivet spacing
Q	rivet force
S	applied stress
t	thickness
v	displacement
$\bar{v}$	relative displacement
w	stringer width
μ	ratio of stringer stiffness to total stiffness

Subscripts:

i	at ith rivet
j	at jth rivet
n	at nth rivet
s	pertaining to stringer
lim	limiting

Superscripts:

bs	broken stringer

FORMULATION OF PROBLEM

Figure 1 shows a stiffened sheet with a crack in the sheet extending equally from opposite sides of a rivet. The stringers and rivets are uniform in size and spacing. One of the stringers that intersect the crack is broken. (The formulation is applicable to any number of broken stringers as long as symmetry is maintained.) The sheet and stringers, which can be of different materials, are subjected to the uniaxial stresses S and SE_s/E respectively, which produce equal strains at large distances from the crack. A state of plane stress is assumed to exist and all forces are assumed to act at the midplane of the sheet.

The stress intensity factor for the sheet and stringer load concentration factor were calculated as in reference 1 except that the equations for the unknown rivet forces were modified to account for the broken stringer. These modifications are shown in the Appendix.

Because the stiffened sheet in figure 1 is infinite in extent, the number of unknown rivet forces is likewise infinite. However, because the remote stresses in the sheet are unaffected by the crack, the remote rivet forces are small and can be neglected. It was found in reference 1 that the calculations were affected by less than 1 percent when they accounted for only those rivets within a rectangular region with a height equal to 2a (or 20 rivet spacings, whichever was larger) and with a width extending beyond the crack tips to include the next stringers (see figure 1).

When stringers are broken, however, the stresses in the sheet are affected over a larger region. For this reason the height of the region in figure 1 was increased to 12a or 20 rivet spacings, whichever was larger. Moreover, the region over which the stresses are affected is larger for stiffer stringers, and becomes infinite for rigid stringers. Consequently, calculations were made only for $\mu \leq 0.7$ where μ is the ratio of the stiffness of the stringers alone to the stiffness of the stringers and sheet. For the uniformly stiffened sheet,

$$\mu = \frac{wt_s E_s}{wt_s E_s + btE} \tag{1}$$

To reduce the number of unknowns, the rivet spacing p beyond the height 2a was increased to 5p. This reduced the number of unknowns by 2/3 (still two times that for all-intact stringers) and reduced the computing time by 90 percent. The effect of this reduction on the accuracy of the calculations was small. The accuracy was checked by increasing the height of the region to 40a and the width to include four additional stringers. The results were found to be affected by less than 5 percent for one broken stringer and less than 10 percent for three broken stringers with $\mu = 0.7$. These errors depended strongly on μ and, for $\mu = 0.5$, were less than one-half those for $\mu = 0.7$. In all cases the stress intensity factor and the force in the most highly loaded stringer that were calculated for the region of height 12a were higher than those calculated for the region of height 40a.

RESULTS AND DISCUSSION

Stress Intensity Factor for the Sheet

Figure 2a shows the stress intensity factor plotted against half-length of the crack for various numbers of broken stringers, M_b. For the case of a single broken stringer, figures 2b - 2d show the effects of stringer stiffness, rivet spacing, and stringer spacing. Because variations in rivet diameter and stringer width have only a small effect, values of $d = p/4$ and $w = 5d$ were used for all the calculations.

Figure 2a shows that the stress intensity factor is higher when the stringers are broken than when all stringers are intact, and increases with the number of broken stringers. When a crack tip is near a broken stringer, the local transfer of load from the broken stringer into the sheet at the first rivet causes the stress intensity factor to be higher than that for an unstiffened sheet. However, when a crack extends beyond the nearest intact stringer, the stress intensity factor is reduced below that of an unstiffened sheet $(K/S\sqrt{\pi a}=1)$. Also, when a crack extends beyond several intact stringers, the effect of the broken stringers become small, especially for a single broken stringer.

Figures 2b and 2c show that, when a crack tip is short of the first intact stringer and is influenced largely by the broken stringer, the stress intensity factor is higher for stiffer stringers and more closely spaced rivets. In this case the stiffer stringers transfer a larger load into the sheet near the crack tip and the more closely spaced rivets transfer the load into the sheet closer to the crack tip. However, for longer cracks, the curves cross and the sequence is inverted.

Figure 2d shows that for all crack lengths, the stress intensity factor is lower for more closely spaced stringers.

Thus, for long cracks when the crack extends beyond the nearest intact stringer, the effect of stringer stiffness, stringer spacing, and rivet spacing is the same with and without broken stringers.

Stringer Load Concentration Factor

When one or more stringers are broken, the pair of intact stringers nearest the center of the panel are the most highly loaded. Figures 3a - 3d show the stringer load concentration factor for the most highly loaded stringer plotted against half-length of the crack for the same variables shown in figures 2a - 2d. The stringer load concentration factor L was defined (ref. 1) as the ratio of the maximum force in the stringer to the remote force applied to the stringer.

L is essentially unity for small cracks and increases with increasing crack length. Figure 3a shows that for long cracks, the curves approach limiting values that increase with the number of broken stringers. In reference 1 it was shown for all intact stringers that L asymptotically approached a limiting value with increasing crack length and that this value was given by

$$L_{lim} = \mu^{-1} \tag{2}$$

Making the simplifying assumption that all of the load in the broken stringers is transferred to the nearest intact stringer on either side of the crack, the limiting value of L is

$$L_{lim} = \mu^{-1} + \frac{1}{2} M_b \tag{3}$$

The values of L_{lim} given by equation (3) are shown on the graph. For long cracks, the calculated values of L approach the appropriate L_{lim}.

Figures 3b - 3d show that L is lower for stiffer stringers and that, for larger values of rivet and stringer spacing, longer cracks are required for L to reach L_{lim}. In these cases also, the calculated values of L approach the appropriate L_{lim}.

Thus, for all crack lengths the situation for broken stringers is much the same as for all stringers intact except that the stringer load concentration factor is increased by $\frac{1}{2} M_b$ due to the broken stringers.

The Effect of Broken Stringers on Residual Strength and Fatigue-Crack-Growth

Because broken stringers affect the stress intensity factor differently for short and long cracks, figure 2 does not suggest a stiffened sheet configuration which is optimum with respect to design criteria for fatigue and fracture. For this reason, calculations were made for the residual strength and fatigue-crack-growth life of a stiffened 2024-T3 aluminum sheet with a broken stringer. The fatigue crack growth and fracture toughness properties in reference 2 were used.

Residual strength

Figure 4 shows the nominal stress that must be applied to the stiffened sheet with one broken stringer to produce $K = K_c$, the critical value. The values are plotted against the half-length of the crack. Curves are shown for various values of stringer stiffness and, for comparison, an unstiffened sheet and a sheet with all stringers intact. Assuming that the sheet will fail before the remaining stringers, the residual strength (defined here as the stress required to fail the sheet) is given by the highest point on a curve to the right of the initial crack length. Note that the peaks of the curves essentially approach a constant height as the crack becomes long.

For $\mu = 0.5$, the curve for one broken stringer approaches that of the curve for all stringers intact. Consequently, the residual strengths of a sheet with a broken stringer and one with all intact stringers will be essentially the same.

The curves for the various values of μ show that, except for very small cracks, the strength of a sheet with a broken stringer is higher with stiffer stringers much as when all stringers are intact. Similarly, the strength can be shown to be higher for more closely spaced rivets and stringers.

In the case where additional stringers would fail before the sheet, the use of stiffer stringers would also increase the residual strength by lowering the stringer stress. However, the use of more closely spaced rivets and stringers could cause some decrease in residual strength because of an increase in stringer stress. Space limitations do not permit this case to be discussed here in detail.

Fatigue-crack-growth life

Figure 5 shows the flights to failure plotted against initial half-length of crack for a stiffened sheet with one broken stringer. Curves are shown for various values of stringer stiffness and, for comparison, an unstiffened sheet and a sheet with all stringers intact. A typical gust-loading spectrum and 1g stress for a transport airplane with a ground-air-ground cycle was assumed. The sheet was assumed to fail before the remaining stringers, and nonlinear load spectrum effects such as retardation were not taken into account. Because the residual strength of the stiffened sheets was not exceeded, failure was defined arbitrarily to be a half-length of crack equal to three times the stringer spacing.

The curves show that, except for very small initial crack lengths, the fatigue-crack-growth life of the sheet with one broken stringer is not significantly less than that of the sheet with all stringers intact and is orders of magnitude greater than that for an unstiffened sheet.

The curves for various values of μ show that the fatigue-crack-growth lives of the sheet with one broken stringer are longer for stiffer stringers, much as when all stringers are intact. This can also be shown for more closely spaced rivets and stringers.

CONCLUSIONS

The stress intensity factor for a stiffened sheet is higher with one or more broken stringers than with all stringers intact, expecially when the crack tip is near a broken stringer. However, if the crack tip extends beyond several intact stringers, the stress intensity factor is not strongly affected by broken stringers and is smaller with the use of stiffer and more closely spaced stringers and smaller rivet spacing.

Broken stringers also cause a higher load concentration in the adjacent stringers. The load concentration is lower for stiffer stringers.

For the case where the sheet fails before the remaining stringers, the residual strengths and fatigue-crack-growth lives for a stiffened 2024-T3 aluminum sheet were only slightly lower with a broken stringer and were much higher than those for an unstiffened sheet. They were also higher for stiffer and more closely spaced stringers and more closely spaced rivets -- much as when all stringers are intact.

REFERENCES

[1] C. C. Poe, Jr., NASA TR R-358 (1971).

[2] C. C. Poe, Jr., "Damage Tolerance in Aircraft Structures," (ASTM STP 486, American Society for Testing and Materials, 1971, pp. 79-97).

APPENDIX

MODIFICATION OF RIVET FORCE EQUATIONS TO ACCOUNT FOR BROKEN STRINGERS

The equations for the unknown rivet forces were obtained by modifying those in reference 1 to account for broken stringers. The notation in figure 6 for three broken stringers is used. Because of symmetry, only those rivets in one quadrant are included.

The displacements of the rivets in the broken stringers can be written

$$v_i^{bs} = \begin{cases} v_1 + \bar{v}_i^{bs} & (i = 1, 2, \ldots .n) \\ v_{n+1} + \bar{v}_i^{bs} & (i = n+1, n+2, \ldots .2n) \end{cases} \tag{4}$$

where v_1 and v_{n+1} are displacements of the sheet at the first rivets and $\bar{v}_i^{bs}$ are displacements of the broken stringers at the ith rivet relative to the first rivets. The displacements $\bar{v}_i^{bs}$ can be written in terms of influence coefficients as

$$\bar{v}_i^{bs} = \begin{cases} \sum\limits_{j=2}^{n} \bar{A}_{ij}^{s} Q_j + \bar{B}_i^{s} SE_s/E & (i = 1, 2, \ldots .n) \\ \sum\limits_{j=n+2}^{2n} \bar{A}_{ij}^{s} Q_j + \bar{B}_i^{s} SE_s/E & (i = n+1, n+2, \ldots .2n) \end{cases} \tag{5}$$

The coefficients in equation (5) can be determined approximately by the superposition of the problems in figure 7. (Notation is shown for the central stringer.) Using the coefficients in reference 1 for a finite width stringer and subtracting the displacements at the first rivets,

$$\bar{A}^s_{ij} = A^s_{ij} - A^s_{i1} - A^s_{1j} + A^s_{11}$$

$$\bar{B}^s_i = B^s_i - wt_s A^s_{i1} - B^s_1 + wt_s A^s_{11} \qquad (i = 1, 2, \ldots n)$$

and for the other stringer

$$\bar{A}^s_{ij} = A^s_{ij} - A^s_{i,n+1} + A^s_{n+1,j} + A^s_{n+1,n+1} \tag{6}$$

$$\bar{B}^s_i = B^s_i - wt_s A^s_{i,n+1} - B^s_{n+1} + wt_s A^s_{n+1,n+1} \quad (i = n+1, n+2, \ldots 2n)$$

Equation (6) satisfies the condition of zero force at the break in the stringers, but not zero normal stress. Equating the displacements as the rivets in the sheet and stringers as in reference 1 and using equation (4) for the broken stringers, the equations for the unknown rivet forces are

$$\sum_{j=2}^{n} (A_{ij} + \bar{A}^s_{ij} - A_{1j} - A_{i1} + A_{11})Q_j + \sum_{j=n+2}^{2n} (A_{ij} - A_{1j} - A_{i,n+1} +$$

$$A_{1,n+1})Q_j + \sum_{j=2n+1}^{N} (A_{ij} - A_{1j})Q_j + \Big[\bar{B}^s_i E_s/E - B_i + B_1 - (A_{i1} - A_{11}$$

$$+ A_{i,n+1} - A_{1,n+1}) wt_s E_s/E\Big] S = 0 \qquad (i = 2, 3, \ldots n)$$

$$\sum_{j=2}^{n} (A_{ij} - A_{n+1,j} - A_{i1} + A_{n+1,1})Q_j + \sum_{j=n+2}^{2n} (A_{ij} + \bar{A}^s_{ij} - A_{n+1,j} -$$

$$A_{i,n+1} + A_{n+1,n+1})Q_j + \sum_{j=2n+1}^{N} (A_{ij} - A_{n+1,j})Q_j + \Big[\bar{B}^s_i E_s/E - B_i + B_{n+1}$$

$$- (A_{i1} - A_{n+1,1} + A_{i,n+1} - A_{n+1,n+1})wt_s E_s/E\Big]S = 0$$

$$(i = n+2,\ n+3,\ \ldots 2n)$$

and

$$\sum_{j=2}^{n} (A_{ij} - A_{i1})Q_j + \sum_{j=n+2}^{2n} (A_{ij} - A_{i,n+1})Q_j + \sum_{j=2n+1}^{N} (A^s_{ij} + A_{ij})Q_j$$

$$+ \Big[B^s_i E_s/E - B_i - (A_{i1} + A_{i,n+1})wt_s E_s/E\Big]S = 0 \tag{7}$$

$$(i = 2n+1,\ 2n+2,\ \ldots N)$$

From equilibrium, the forces in the first rivets of the broken stringers are

$$\sum_{j=1}^{n} Q_j + SE_s wt_s/E = 0 \qquad (i = 1)$$

$$\sum_{j=n+1}^{2n} Q_j + SE_s wt_s/E = 0 \qquad (i = n+1) \tag{8}$$

Equations (7) and (8) can be easily extended to any other number of broken stringers.

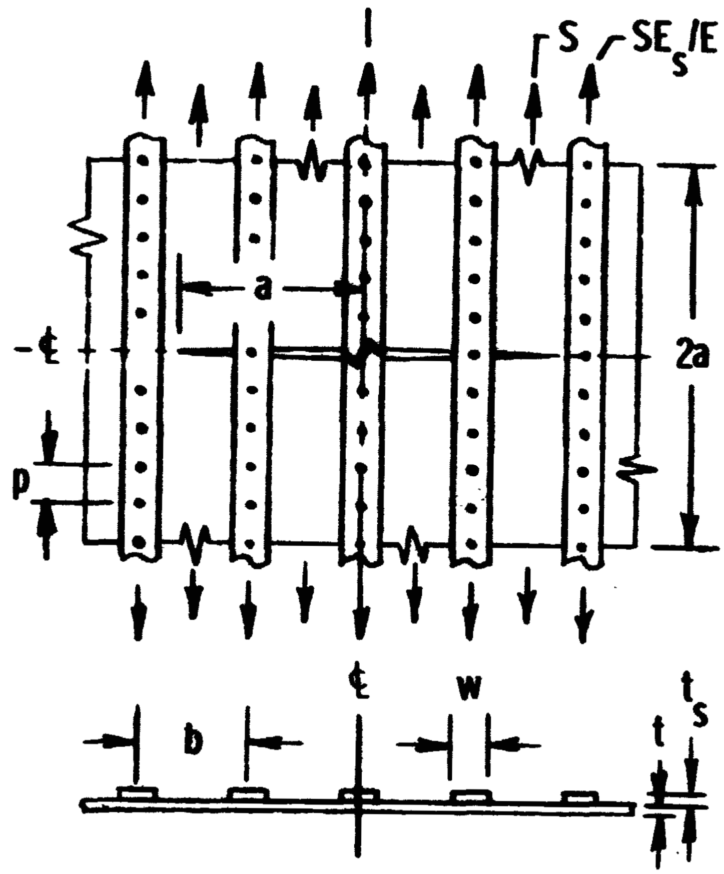

Figure 1.- Cracked sheet with riveted and uniformly spaced stringers.

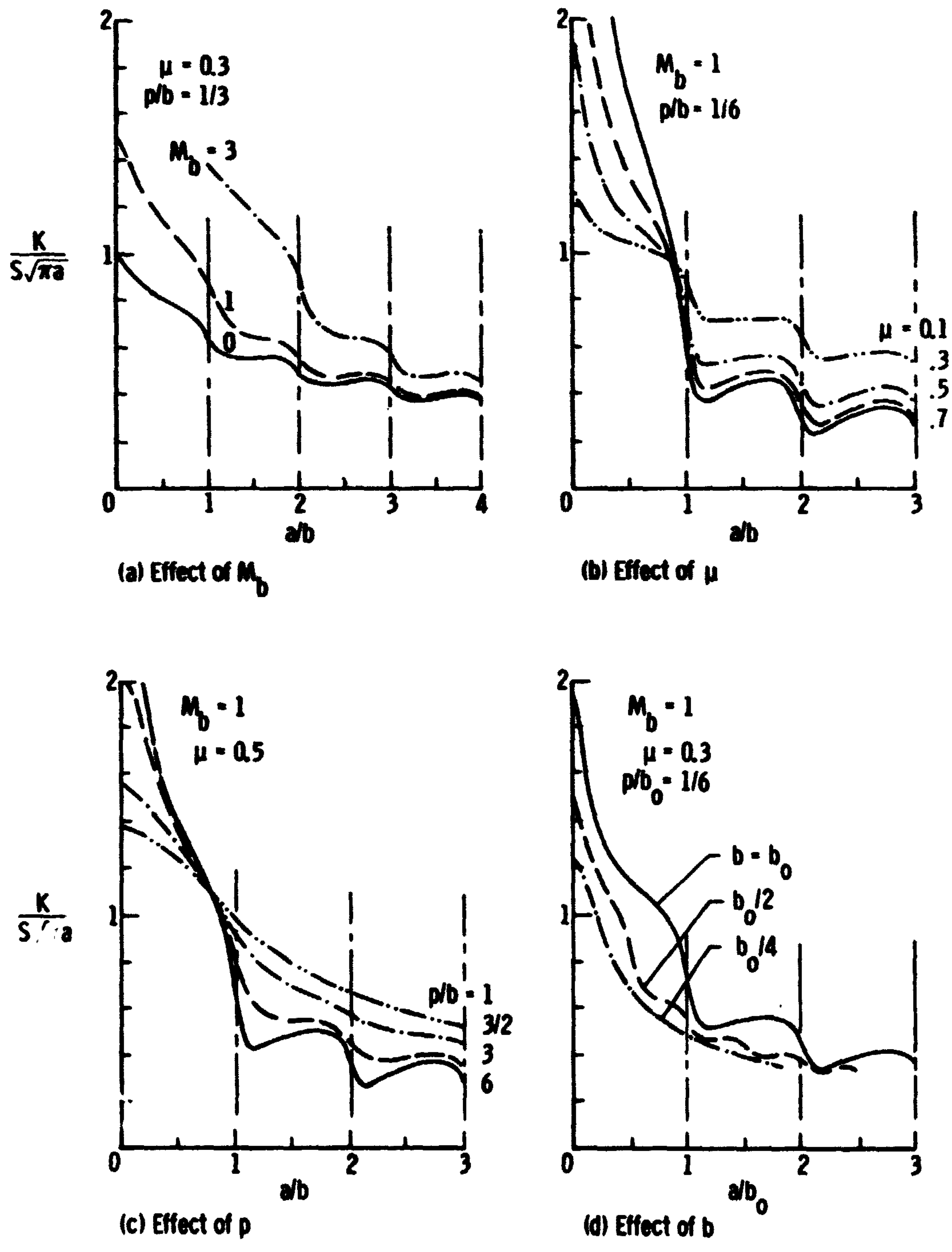

Figure 2.- Stress intensity factor for sheet with broken stringers.

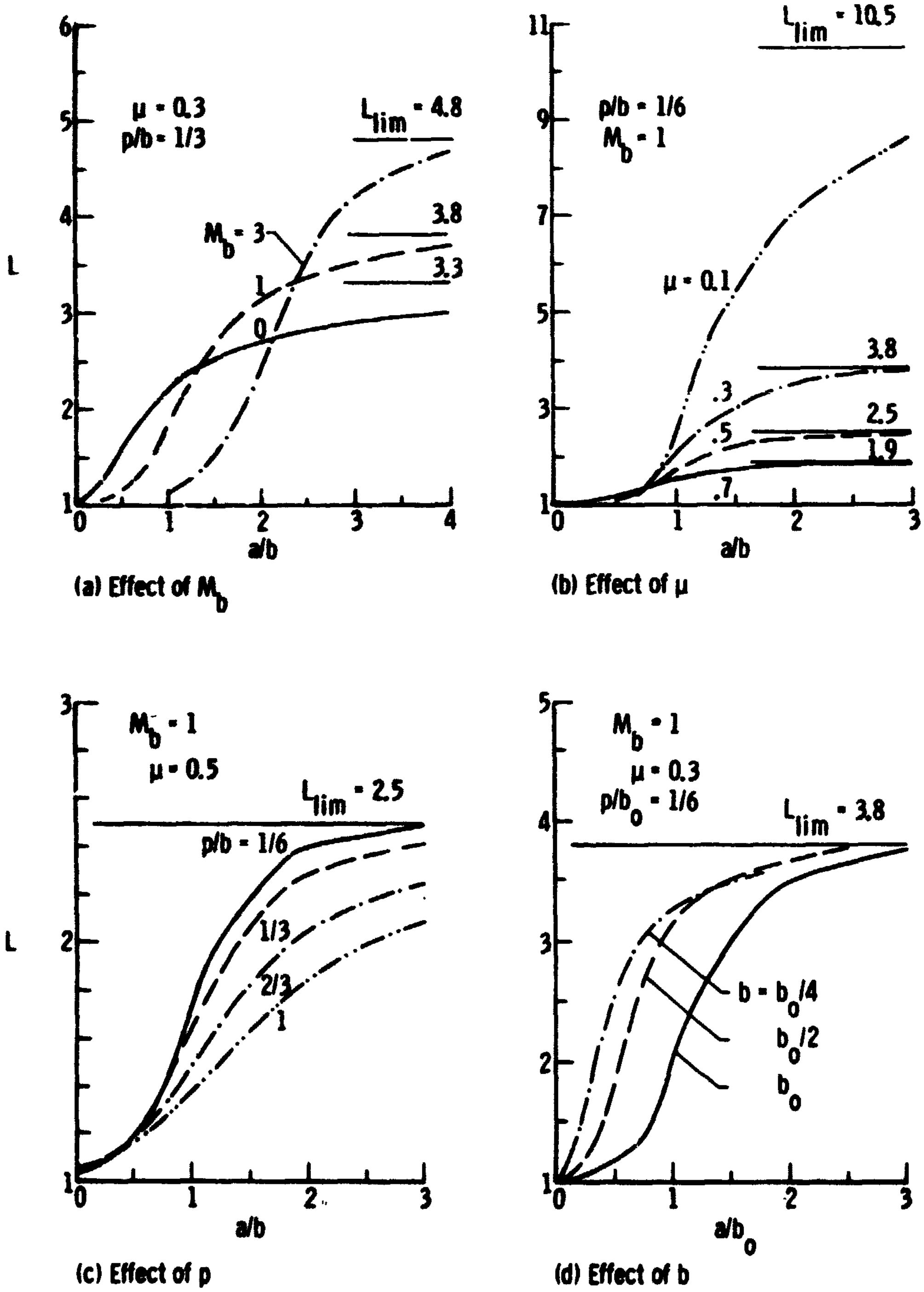

Figure 3.- Stringer load concentration factor for most highly loaded stringer.

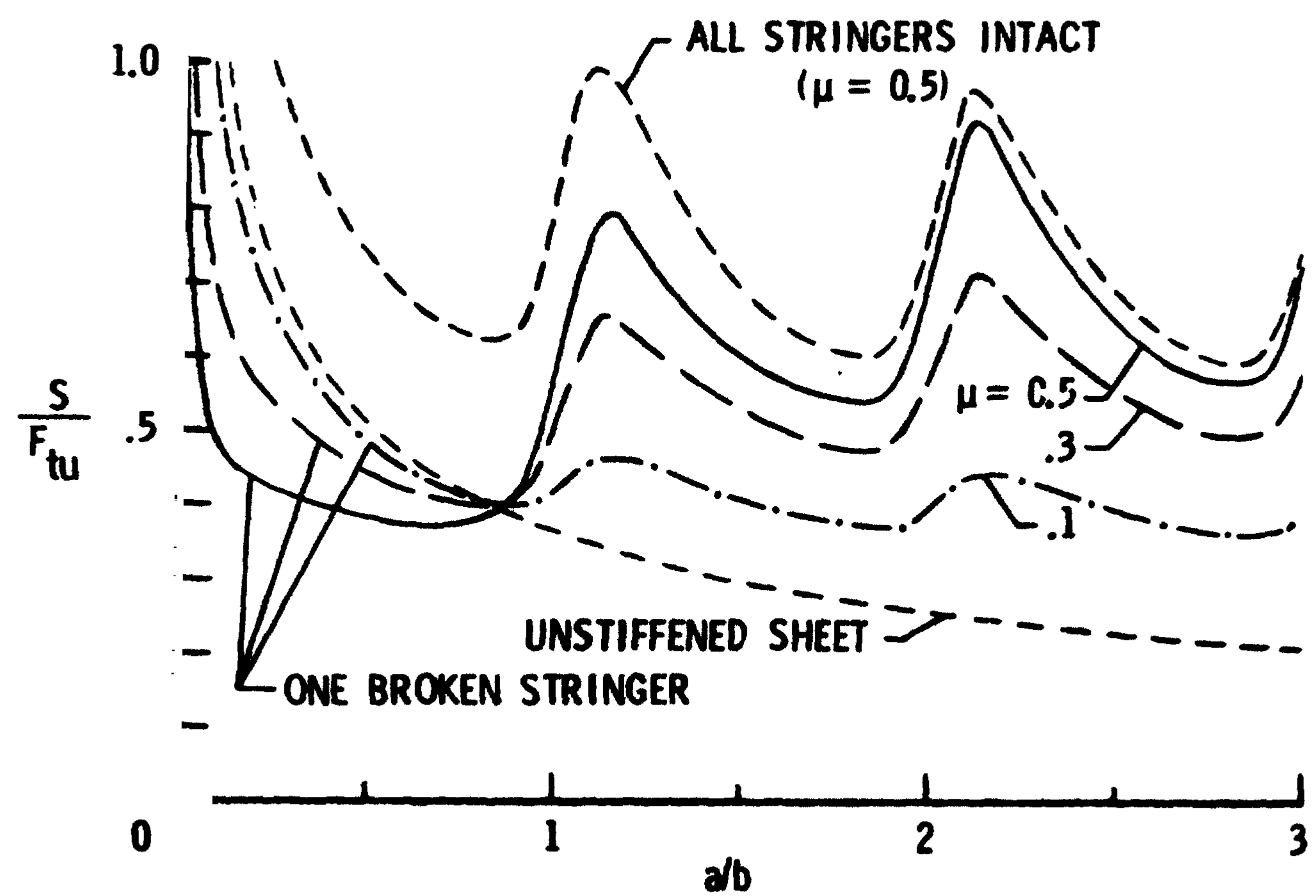

Figure 4.- Residual strength of a stiffened 2024-T3 aluminum sheet with one broken stringer (b=100mm and p=b/6).

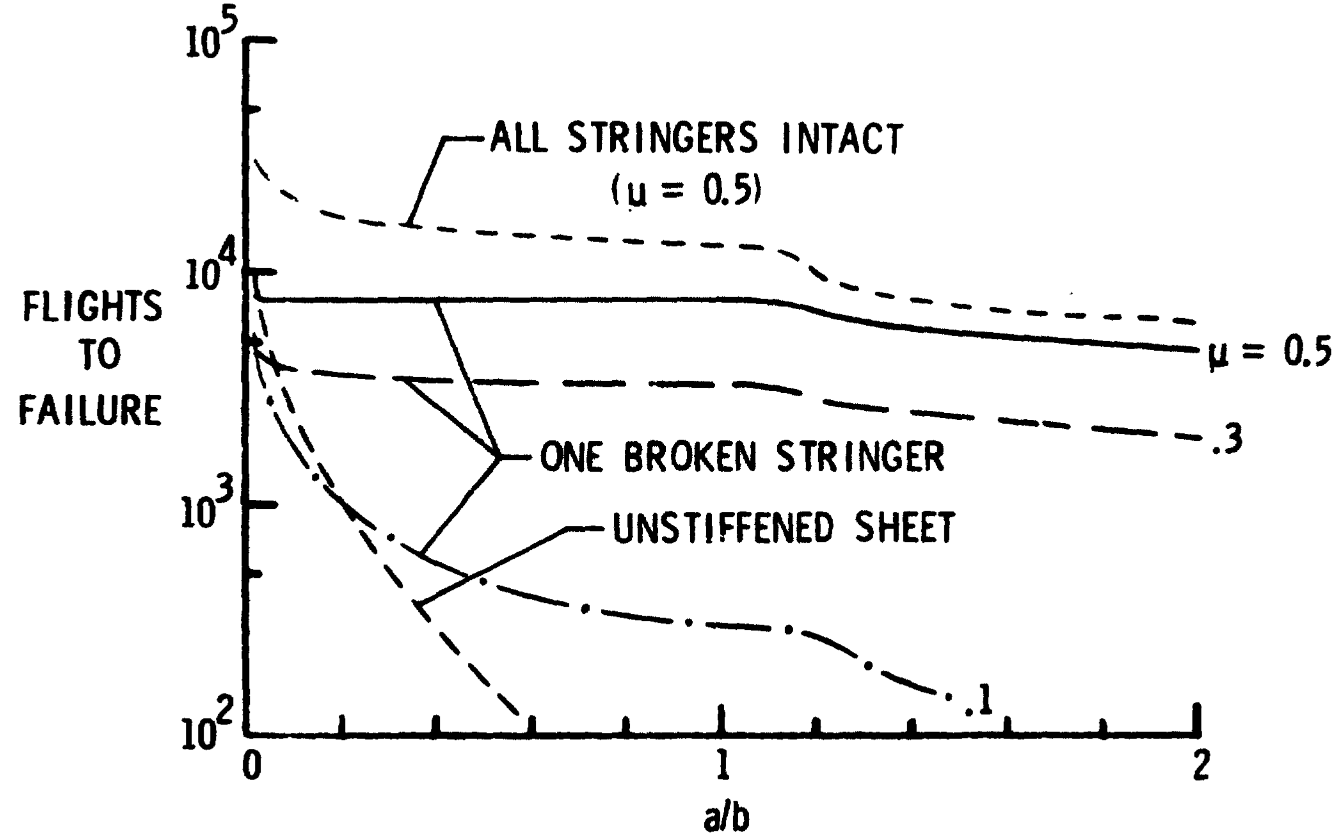

Figure 5.- Fatigue-crack-growth life of a stiffened 2024-T3 aluminum sheet with a broken stringer (b=100mm and p=b/6).

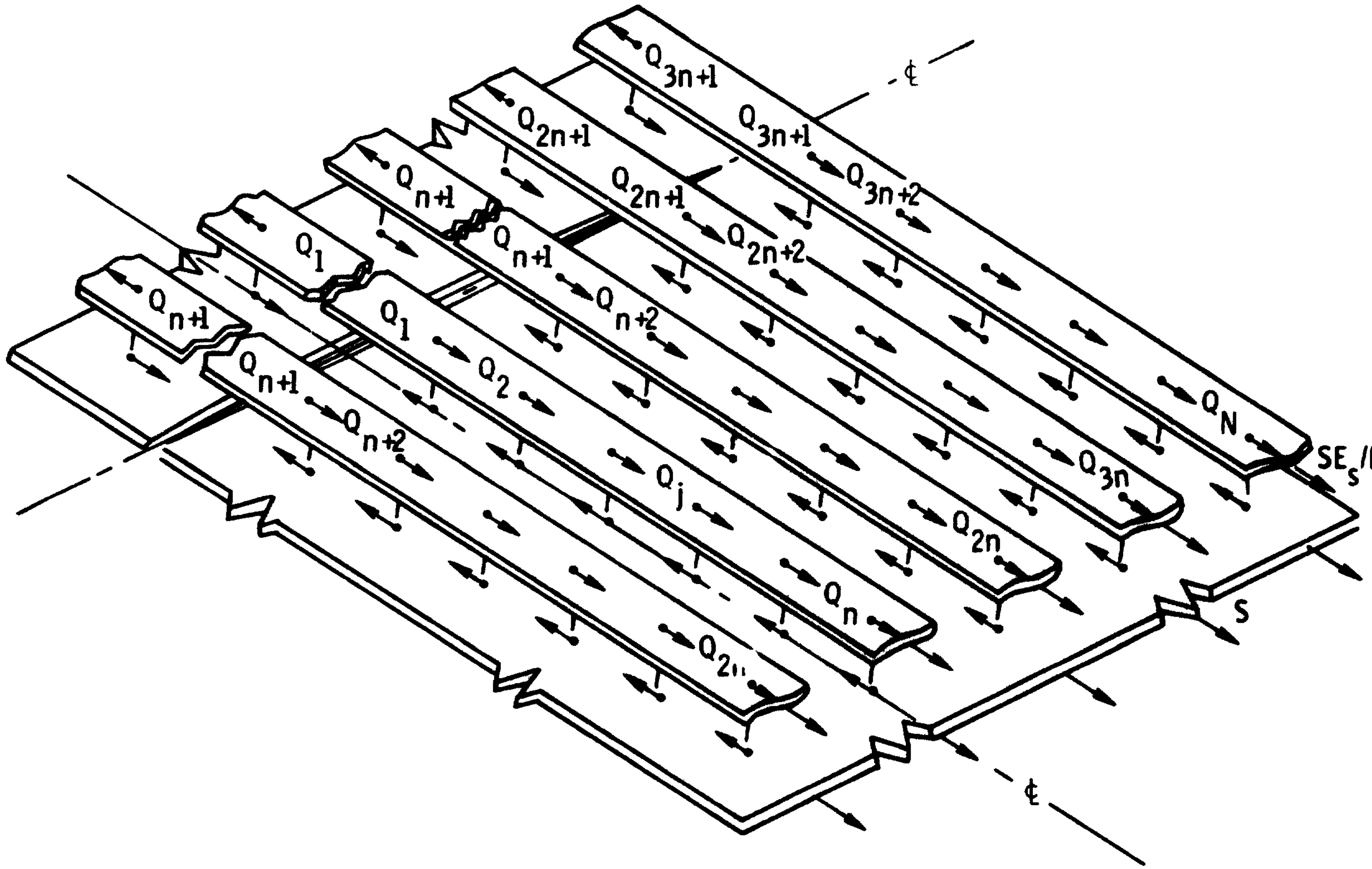

Figure 6.- Notation for rivet forces.

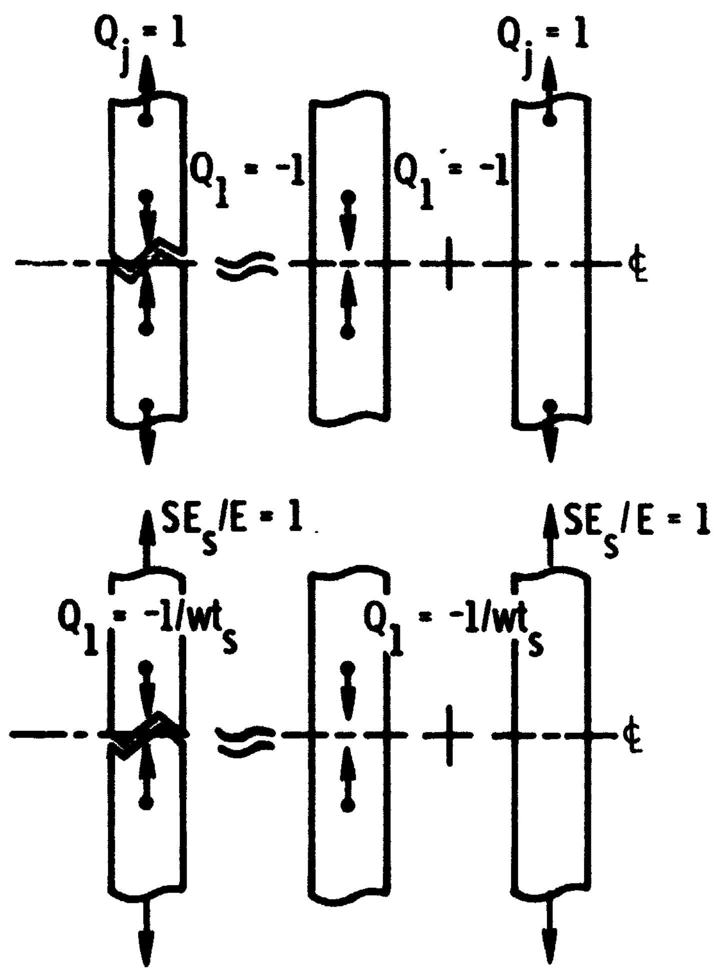

Figure 7.- Superpositions of problems for displacements of a broken stringer.